“School in Deformation Theory IV”
Roma, 16-20 September 2024

Programme

Time	*Monday 16*	*Tuesday 17*	*Wednesday 18*	*Thursday 19*	*Friday 20*
9:00 – 9:55		**Manolache 1**	**Ilten 2**	**Meazzini 3**	**Manolache 4**
10:00 – 10:30	Registration	*Break*	*Break*	*Break*	*Break*
10:30– 11:25	**Meazzini 1**	**Ilten 1**	**Wemyss 3**	**Manolache 3**	**Meazzini 4**
11:30- 12:25	**Wemyss 1**	**Wemyss 2**	**Manolache 2**	**Ilten 3**	**Ilten 4**
2:30 – 3:25	**Lehmann** (start 2:45)	**Meazzini 2**		**Wemyss 4**	
3:30 – 4:00	*Break*	*Break*		*Break*	
4:00 – 4:40	**mini talks** (to 5:30)	**Felten**		Free discussion	
4:45 – 6:15		**mini talks**			

Courses:

Nathan Ilten (Simon Fraser University)

"Deformation theory, computations, and toric geometry"

Abstract: Schlessinger's theorem guarantees that the functor Def_X of infinitesimal deformations of a complete variety X has a hull (also known as a miniversal deformation). However, in practice it is often quite challenging to compute the hull of Def_X. In this series of lectures, we will focus on computing a hull of Def_X in the special case when X is a smooth complete variety. We will begin in the first lecture with some generalities on deformation theory. In the second lecture, we will discuss in detail the well-known fact that deformations of smooth X are controlled by the Cech complex of the tangent bundle. Using this, we will see in the third lecture how to construct a hull of Def_X (at least in theory) by iteratively solving a "deformation equation". Finally, in the fourth lecture we will see how when X is a toric variety,

the problem of constructing a hull of Def_X is almost entirely combinatorial and much more tractable. I will assume no prior knowledge of either deformation theory or toric geometry. The fourth lecture is based on joint work-in-progress with Sharon Robins.

Cristina Manolache (University of Sheffield)

"Reduced invariants"

Abstract: The main object of interest will be curve counts in certain varieties. Deformation theory will allow us to understand moduli spaces of such curves. I will start by explaining why familiar moduli spaces of such curves are reducible and how to obtain better spaces. I first explain a blow-up of genus one curves in projective spaces (due to Zinger, Li--Zinger, Vakil--Zinger) and I will continue with an all genus construction obtained by blowing up sheaves (Cobos Rabano--Mann--M--Picciotto). While the resulting blow-ups are geometrically natural, they have no obvious modular interpretation. I will discuss a modular interpretation of the Vakil--Zinger genus one blow-up via log geometry (due to Ranganathan--Santo-Parker--Wise). I will end by discussing intersection numbers on these blown-up spaces and open questions.

Francesco Meazzini (Università degli Studi di Roma "La Sapienza")

"An introduction to deformations of sheaves"

Abstract: We will introduce the basic notions concerning functors of Artin rings and we will discus the fundamental example of deformation functor associated to a differential graded Lie algebra. These tools will be then applied to the study of infinitesimal deformations of sheaves.

Michael Wemyss (University of Glasgow)

"Noncommutative Deformations and Classification Problems"

Abstract: This mini-course will be an introduction to noncommutative deformation theory, and will also outline some of its applications to algebraic geometry. The main difference when we add the adjective "noncommutative" to deformation theory is that we make the category of test objects larger, compared to the classical case. By doing this, we get more information. The first lecture will outline how to formulate noncommutative deformation theory using naive functors, and will give some general results. The second lecture will outline how to do this using DGAs. There are two key points: (a) we really do need a DGA to be able to noncommutatively deform; a DGLA does not suffice, and (b) deforming multiple objects is now also possible. The third and fourth lectures will explain how this works when we apply the above theory to curves within 3-folds, where noncommutative deformations turn out to be the classifying object.

Talks:

Simon Felten (Columbia University)

"Curved Lie algebras in logarithmic deformation theory"

Abstract: Every classical deformation problem in characteristic 0 is controlled by a dg Lie algebra. This is no longer true in logarithmic geometry, where the infinitesimal deformation functor of a log smooth variety cannot always be controlled by a dg Lie algebra. In this talk, we will see how allowing curvature in the dg Lie algebra remedies this situation.

Alessandro Lehmann (University of Antwerp and SISSA)

"Curved deformations of differential graded algebras"

Abstract: It is a classical fact that the Hochschild complex of an algebra governs its deformations as an associative algebra. Surprisingly this does not immediately generalize to differential graded algebras (and by extension, categories), since in general deformations given by Hochschild cocycles can introduce curvature. In this talk I'll explain how to make sense of these curved deformations via a novel notion of a type of derived category; I will also explain how this construction suggests a new notion of deformation of a triangulated category, based on categorical square zero extensions. This is based on joint work with Wendy Lowen.

Mini Talks:

Filippo Belfiori (Università di Bologna)

"An example of obstructed surface"

Abstract: TBA

Nicolò Bignami (SISSA)

"On the Beauville-Voisin conjecture for Hilbert schemes of points on a K3 surface"

Abstract: The Beauville-Voisin conjecture for a hyperkähler variety X states that the subring of the Chow ring of X generated by the divisor classes and Chern characters of the tangent bundle injects into the cohomology ring of X.

In this mini talk, I will explain how Negut and Maulik addressed a slightly weaker version of this conjecture in the case of the Hilbert scheme of points on a K3 surface.
Their proof involves studying the geometry of nested Hilbert schemes on the surface to derive formulas which involve characteristic classes of tautological sheaves and to package the relations obtained in Chow in the language of representation theory.

Luca Dal Molin (Università di Trento)

"*A Primer on the Moduli Stack of Real Vector Bundles over Curves*"

Abstract: One of the most studied problems in moduli theory is the study of vector bundles over fixed algebraic varieties. The scope of my PhD project is to study the particular case of the moduli stack of vector bundles over a real algebraic curve. In this small talk, we will discuss the fundamental objects used in this specific moduli problem and outline the planned research steps along with the original works that inspired the topic.

Federico Fallucca (Università di Milano Bicocca)

"*New results on the degree of the canonical map of surfaces of general type*"

Abstract: It is a well-known fact that the canonical map of a curve C of genus at least two is either an embedding or of degree 2. The latter happens if and only if C is hyperelliptic. For smooth surfaces S of general type the situation is more difficult.
Persson proved in 1978 that the degree of the canonical map of S is bounded from above by 36 and Beauville showed that surfaces with a high degree of the canonical map (>27) have q=0 and pg=3. For a long time only Persson example of degree 16 and Tan example of degree 12 had been discovered until Rito and Yeung constructed in 2019 an example of degree 36, proving Persson's bound is sharp.
As a consequence of this, Mendes Lopes and Pardini revived the topic on the degree of the canonical map and posed in a recent survey the natural question:
For every d\leq 36, does there exist any surface S of general type with pg=3, q=0, and canonical map of degree d?
In this seminar, we briefly present the first examples in the literature to our knowledge of surfaces with a canonical map of degree d=10,11,14 (joint work Gleissner) and d=13,15, and 18.

Christian David Forero Pulido (SISSA)

"*Hall Algebras for Toric Monoid Schemes*"

Abstract: The formalism of Hall algebras was invented only after some motivating examples were discovered. As a matter of fact, Hall algebras appear in different areas of mathematics, such as modular or p-adic representation theory (in the form of the functors of parabolic induction/restriction), number theory and automorphic forms (Eisenstein series for function fields), and in the theory of symmetric functions. They are associative algebras with a basis corresponding to isomorphism classes of some geometric object, for instance coherent sheaf over a variety. However, Hall algebras for coherent sheaves on higher-dimensional varieties are hardly understood.

In this talk, I will present a novel approach primarily developed by Matt Szczesny, which extends the theory of Hall algebras into the realm of the field with one element, F1, as a playground to gain more inside . The discussion will focus on monoid schemes, particularly toric monoid schemes, which provide a more accessible and elegant context for these algebraic structures due to their connection with combinatorial structures.

I will begin with a brief overview of monoid schemes, including the theory of coherent sheaves over them, and then demonstrate how Hall algebras can be explicitly constructed in this context. This approach not only elucidates the complexities of Hall algebras but also offers concrete examples analogous to those in classical settings.

Alessandro Frassineti (Università di Genova)

"Birational geometry of the moduli space of pointed K3 surfaces"

Abstract: K3 surfaces are among the most studied classes of algebraic varieties, being important both from the historical point of view and for applications. In this talk, I will recall the famous "Mukai models" list for K3 surfaces and how they relate to the birational type of the moduli space of K3 surfaces. Next, we will see the generalisation to the "pointed case", i.e. where the moduli space parametrises K3 surfaces with a bunch of fixed points over them.

Soumik Ghosh (Yale University)

TBA

Abstract:

Yijue Hu (University of Nottingham)

"K-stability of X_{2,n}"

Abstract: K-stability originates from differential geometry, to describe the existence of K\ddot{a}hler-Einstein metric on a complex Fano manifold. The notion was introduced by Tian and later Donaldson put it into the language of algebraic geometry. I'll shortly introduce what K-stability is and talk about the Abban-Zhuang method on determining K-stability of Fano varieties. We hope to apply Abban-Zhuang to X_{2,n}, which is, the complete intersection of a quadric and a degree n hypersurface in \mathbb{P}^{n+2}.

Dongchen Jiao (Brunel University)

"Q-complements for Fano foliations"

Abstract: We investigate the existence of complements for algebraically integrable Fano triples. For classical Fano pairs (X, B) (and even for generalised Fano pairs (X, B+M), Birkar proved the existence of bounded n-complements under suitable singularity conditions. Therefore, it would be natural to ask the same question for Fano foliations and furthermore, for Fano triples. However, the first problem is that we do not even have the existence of Q-complements due to the failure of Bertini's type theorem for foliations. To solve this, we use the recently developed MMP for foliations to prove that generic log canonical implies global log canonical for log Calabi-Yau foliated triples. This is a joint work with Y. Chen and P. Voegtli.

Leandro Meier (University of Jena)

"Minimal log discrepancy and complexity one T-varieties"

Abstract: We consider Shokurov's conjecture regarding boundedness of minimal log discrepancies. We briefly recall some previous results and show that the conjecture holds for complexity one Q-factorial Fano cone singularities. This is based on joint work with Hendrik Süß.

Matteo Montagnani (SISSA)

"Smooth and proper categories in analytic geometry"

Abstract: After reviewing the definitions of smooth and proper categories and their role in algebraic geometry, I will discuss a theorem by Toën and Vaquié that demonstrates why this concept is inherently algebraic and not suitable for the complex analytic setting. Finally, we will explore how to extend this result to the non-Archimedean and formal settings.

Torger John Olson (University of Oslo)

"The McKay correspondence"

Abstract: One version of the McKay correspondence states that for a finite subgroup G of SL(2,$\mathbb{C}$) there is an isomorphism between the G-equivariant K-theory of $\mathbb{C}^2$ and the ordinary K-theory of the desingularization, X, of the quotient $\mathbb{C}^2$/G: the McKay quiver of G coincides with the dual graph of the exceptional divisor in X. Bridgeland--King--Reid have shown that this comes from an equivalence of derived categories.

Filippo Papallo (Università di Genova)

"An approach to non-commutative algebraic geometry"

Abstract: "To do geometry you do not need a space, you only need an algebra of functions on this would-be space". This quote by Manin embodies the idea behind many reconstruction theorems, for instance the one by Gelfand-Niemark, which unveils the duality between commutative C*-algebras and locally compact Hausdorff spaces. While looking for a consistent quantum model, physicists started to wonder what would be the geometric counterpart to a non-commutative algebra; in algebraic terms, does Spec exists for non-commutative rings? This mini-talk will introduce some ideas in non-commutative algebraic geometry.

Prajwal Samal (IMPAN)

"Gorenstien Calabi-Yau Varieties and Mirror Symmetry"

Abstract: Gorenstien Calabi-Yau varieties are Calabi-Yau varieties with an arithmetically Gorenstien embedding in a projective space. Such an embedding can be achieved for any Calabi-Yau. I will give the status of classification of such Calabi-Yau 3-folds and our ongoing work towards their construction in codim 5. I will also present our ongoing work towards constructing the mirrors to such varieties using various techniques like deformation theory, toric geometry etc.

Julie Symons (University of Antwerp)

"An equivalence of DG-derived deformations"

Abstract: In ongoing joint work with Wendy Lowen, Michel Van den Bergh and Francesco Genovese, we aim to further develop and understand the deformation theory of pretriangulated dg-categories with a sufficiently nice t-structure as introduced in [GLV21; GLV22]. Our main theorems establish an equivalence of deformation pseudofunctors, relating so-called t-deformations of such a (bounded) pretriangulated dg-category A to classic dg-deformations of the full dg-subcategory of derived injectives DGInj(h-proj(A)) of h-proj(A) (these can be regarded as the building blocks, much like the injective objects in the abelian setup). Pointwise, this amounts to an equivalence

(1) Def^t_A(θ) ≅ Def^dg_{DG-Inj(h-proj(A))}(θ),

where θ : R → S is a base change morphism of dg-rings. This would then be a dg-derived analogue of the equivalence that was established in [LV06] in the abelian setting. I will commence the talk by discussing our motivation for (1), which stems from the fact that it provides a deformation-theoretic interpretation^1 of the higher Hochschild Cohomology groups HHn (A), n ≥ 3, of an abelian category A. Next, I will outline our approach, drawing parallels with the abelian story. Since one direction of the

equivalence (1) has been addressed in [GLV22] using the reconstruction theorems of [GLV21] – namely that a dg-deformation of the dg-category of derived injectives induces a t-deformation between the pretriangulated t-dg-categories – I will focus on the converse. The aim is to provide an overview of the proof and its various components and mention the technical issues we are still facing. I will treat one key component in more detail: a dg-enhancement of the derived category D(A) of a pretriangulated dg-category in terms of filtered homotopy colimits. This enhancement allows us to extend t-structures from A to D(A).
References

[GLV21] F. Genovese, W. Lowen, and M. Van den Bergh, "T-structures and twisted complexes on derived injectives," Adv. Math., vol. 387, Paper No. 107826, 70, 2021.

[GLV22] F. Genovese, W. Lowen, and M. Van den Bergh, T-structures on dg- categories and derived deformations, 2022. arXiv: 2212.12564 [math.CT].

[LV06] W. Lowen and M. Van den Bergh, "Deformation theory of abelian categories," Trans. Amer. Math. Soc., vol. 358, no. 12, pp. 5441–5483, 2006.

^1 Recall that there is an interpretation of HH^{2,3} (A) in terms of flat infinitesimal abelian deformations and obstructions thereof when extending to formal ones.

Oliver Sokvari (Humboldt University of Berlin)

"Simple surface singularities"

Abstract: Simple hypersurface singularities were classified by Arnol'd. In the two-dimensional case these are exactly the Kleinian singularities, and their deformation theory can be understood by the corresponding Dynkin diagrams. Simple surface singularities in general are much more mysterious with plenty of open questions.

Xiaoxiang Zhou (Humboldt University)

"Six-Functor Formalism and Tanakian Formalism"

Abstract: In this talk, we will rapidly introduce the traditional six-functor formalism. Following this, we will discuss the convolution structure on perverse sheaves over an Abelian variety and relate it to representations of algebraic groups.