Winter School "Workshop in Deformation Theory III"

Bari, 19-23 February 2018

Programme

Time	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
9:30 – 10:45		**Pacienza 1**	**Martinengo 3**	**Lehn 3**	**Markl 3**
10:45 – 11:15		*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*
11:15 – 12:30		**Martinengo 2**	**Lehn 2**	**Markl 2**	**Pacienza 3**
14:00 – 14:30	*Registration*				
14:30 – 15:45	**Martinengo 1**	**Markl 1**		**Pacienza 2**	
15:45 – 16:15	*Coffee break*	*Coffee break*		*Coffee break*	
16:15 – 16:55	**Lehn 1** (16:15-	**Carocci**		**Lefèvre**	
17:00 – 17:40	**Lehn 1** 17:30)	**Cattaneo**		**Weber**	

Courses:

Manfred Lehn (Johannes Gutenberg Universität Mainz)

"Moduli of Rational curves"

Abstract: I would start with discussing properties of Hilbert schemes and Grassmannians, pass to moduli of conics and twisted cubics, and discuss moduli of such curves on cubic varieties, including of course problems of deformation theory arising in this context.

Martin Markl (Mathematical Institute of the Czech Academy of Science)

"Deformations of algebras and their diagrams"

Abstract: My series of talks will be devoted to deformation theory of algebras and their diagrams. After recalling the classical Gerstenhaber's theory for associative algebras, I will shift our attention to homotopy invariant setup of Maurer-Cartan moduli spaces. As an application, I review Kontsevich's approach to deformation quantization of Poisson manifolds. If time permits (which will probably not be the case), after a brief introduction to operads, I will describe L-infinity algebras governing deformations of (diagrams of) algebras of a given type and give an example of such an algebra.

My talks will be based upon the monograph.

M. Markl: Deformation Theory of Algebras and Their Diagrams, CBMS, Regional Conference Series in Mathematics, volume 116. Published by the American Mathematical Society, Providence, Rhode Island, 2012.

Elena Martinengo (Università degli Studi di Torino)

"Introduction to deformation theory"

Abstract: In the first lecture I will present the functorial approach to deformation theory: functors of Artin rings and deformation functors, their prorappresentability, smoothness of a natural transformation of functors and obstruction theory.

The second lecture will be devoted to deformations of affine schemes: in particular, to the description of first order deformations and obstructions.

In the third lecture (depending on the time left) I will present some examples and introduce the dglas approach to deformation theory.

Gianluca Pacienza (Institut Èlie Cartan de Lorraine)

"Deformation theory of isotropic subvarieties of irreducible holomorphic symplectic varieties"

Abstract: In the lectures I will first review the basic material on irreducible holomorphic symplectic (IHS) varieties, and then turn to the study of the deformation theory of "special" subvarieties of IHS varieties. The guiding example will be that of a lagrangian subvariety, that has been treated several years ago by Claire Voisin. We will discuss it in detail and then concentrate on (hopefully!) a couple of recent generalizations (obtained by Christian Lehn and myself and by Daisuke Matsushita).

Talks:

Francesca Carocci (Imperial College)

"*Reduced vs Cuspidal GW invariants for the quintic 3-fold*"

Abstract: Moduli spaces of stable maps of genus g>0 are highly singular and with many irreducible components which affect the enumerative meaning of the invariants arising from them. In this brief talk we will try to give a flavour of how bad these spaces can be, already in the simplest example in genus 1. We will then hint at two possible approaches to deal with the so called "degenerate contributions" and state in which sense these two ways coincide for the quintic 3-fold.

Alberto Cattaneo (Università di Milano\Université de Poitiers)

"Automorphisms on deformation families of irreducible holomorphic symplectic manifolds"

Abstract: Boissière, Camere and Sarti provided a classification of non-symplectic automorphisms of prime order acting on irreducible holomorphic symplectic fourfolds deformation equivalent to the Hilbert scheme of two points on a K3 surface. This classification relies on the study of the invariant lattice of the automorphism (and its orthogonal complement) inside the second cohomology group with integer coefficients, equipped with the Beauville-Bogomolov-Fujiki quadratic form. I will report on a joint work

with Chiara Camere about extending this classification to manifolds of higher dimension, which are still deformations of Hilbert schemes of points on K3 surfaces. I will also discuss some explicit ways to produce automorphisms of such varieties.

Louis-Clément Lefèvre (Institut Fourier):

"Mixed Hodge theory and deformations of representations of fundamental groups of complex algebraic varieties"

Abstract: The mixed Hodge theory of Deligne gives additional structures on the cohomology of complex algebraic varieties. Via rational homotopy theory, mixed Hodge structures have been constructed on homotopy groups of such varieties by J. Morgan and R. Hain. In this vein, we construct mixed Hodge structures on certain invariants associated to linear representations of fundamental groups of complex algebraic varieties. The starting point is the theory of Goldman and Millson that relates the study of deformations of representations of fundamental groups to the deformation theory via differential graded Lie algebras. We review these ideas using L-infinity algebras and a construction due to D. Fiorenza and M. Manetti of which we show some compatibility with mixed Hodge theory. This allows us to re-write completely and recover a construction of P. Eyssidieux and C. Simpson and to extend their result to some non-compact case.

Thomas Weber (Università di Napoli Federico II):

"Drinfel'd Twist Deformation Quantization on Symplectic Manifolds"

Abstract: If a Lie algebra acts on a Poisson manifold via derivations, the problem of deformation quantization of the manifold can be shifted to the existence of a normalized 2-cocycle on the universal enveloping algebra. This process is called Drinfel'd twist deformation quantization. Unlike the general Kontsevich star product, a twist star product inhabits many additional properties which are of particular interest in physics. However, it is the task of this talk to show that the twisting procedure is limited, i.e. that not every deformation quantization is of this form. After introducing the general construction, topological obstructions of twist star products are given in the symplectic case. Moreover, their existence is connected to equivariant line bundles with non-trivial Chern class, using Morita theory. Several examples and counterexamples are presented. Based on joint works with P. Bieliavsky, F. D'Andrea, C. Esposito, S. Waldmann.